10.1

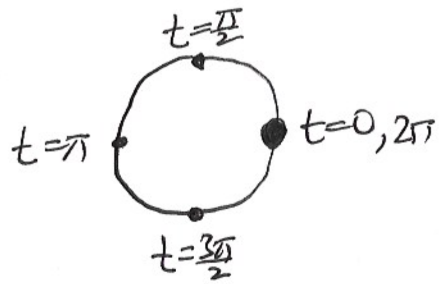
CIRCLE CENTER (h,k) RADIUS r

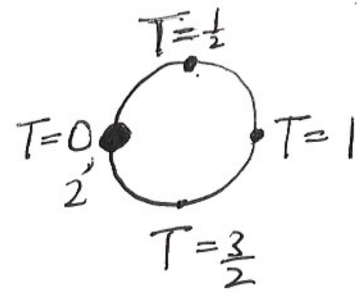
$x = h + r\cos t$
$y = k + r\sin t$
$t \in [0, 2\pi]$

START @ RIGHTMOST POINT
COUNTERCLOCKWISE
ONCE AROUND

CIRCLE CENTER $(-3,2)$ RADIUS 5

START @ LEFTMOST POINT
CLOCKWISE
TWICE AROUND IN 4 UNITS OF TIME
IE. ONCE AROUND IN 2 UNITS OF TIME
THEN DOUBLE TIME





t	π	$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$	$-\pi$
T	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2

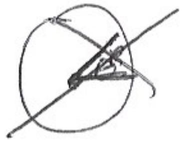
$t = mT + b$ $\quad m = \frac{\Delta t}{\Delta T} = \frac{0-\pi}{1-0} = -$

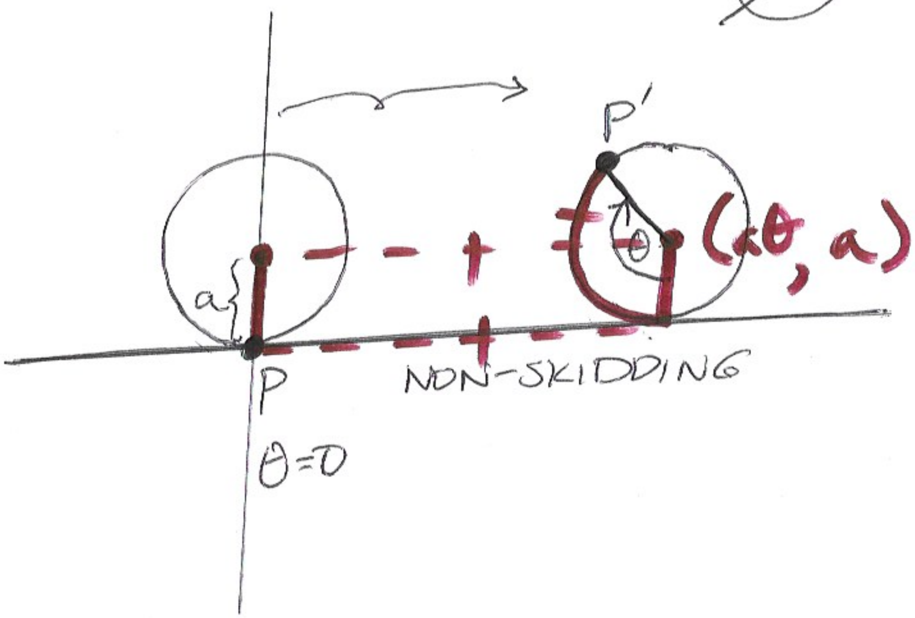
$t = -\pi T + \pi$ $\quad b =$ VALUE OF t WHEN $T = 0$

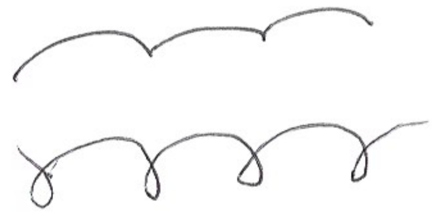
$t = \pi(1-T)$

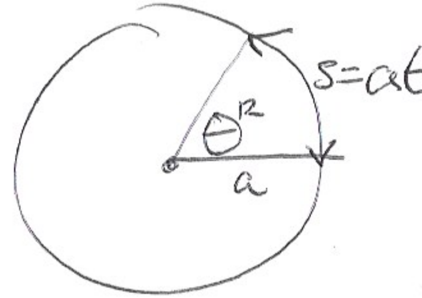
$x = -3 + 5\cos(\pi(1-T))$
$y = 2 + 5\sin(\pi(1-T))$ $\quad T \in [0,4]$

CYCLOID







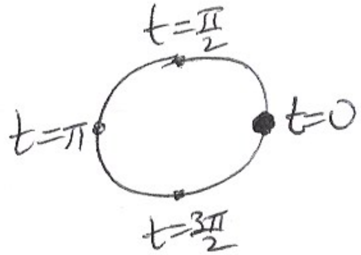


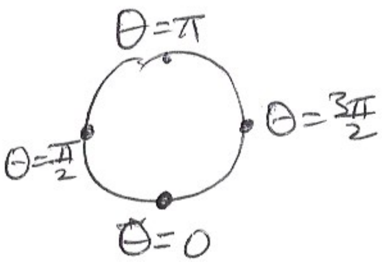
"CIRCLE" CENTER $(a\theta, a)$

RADIUS a

START @ BOTTOM

CLOCKWISE





$$x = a\theta + a\cos\left(\frac{3\pi}{2} - \theta\right)$$

$$y = a + a\sin\left(\frac{3\pi}{2} - \theta\right)$$

$$t = \frac{3\pi}{2} - \theta$$

$$\cos\left(\frac{3\pi}{2} - \theta\right) = \overset{0}{\cancel{\cos\frac{3\pi}{2}}}\cos\theta + \overset{-1}{\cancel{\sin\frac{3\pi}{2}}}\sin\theta$$

$$= -\sin\theta$$

$$\sin\left(\frac{3\pi}{2} - \theta\right) = \overset{-1}{\cancel{\sin\frac{3\pi}{2}}}\cos\theta - \overset{0}{\cancel{\cos\frac{3\pi}{2}}}\sin\theta$$

$$= -\cos\theta$$

$$x = a(\theta - \sin\theta)$$

$$y = a(1 - \cos\theta)$$

10.2 CALCULUS OF PARAMETRIC CURVES

GIVEN $x=f(t)$ $\quad f'(t)=\frac{dx}{dt}$

$y=g(t)$ $\quad g'(t)=\frac{dy}{dt}$

CHAIN RULE

$$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$$

SLOPE OF TANGENT LINE $=\dfrac{dy}{dx}$

$$\frac{dy}{dt}=\frac{dy}{dx}\frac{dx}{dt}$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{dy}{dx}=\frac{g'(t)}{f'(t)}$$

eg. FIND EQ'NS OF T.L. TO $x=4t-t^3$ AT y-INTERCEPTS
$y=t^2-1$

$x=0=4t-t^3=t(4-t^2)$

$t=0,2,-2$

$y=-1,3,3$

$(0,-1)$
$(0,3)$
$(0,3)$

$$\frac{dy}{dx}=\frac{(t^2-1)'}{(4t-t^3)'}=\frac{2t}{4-3t^2}$$

$$\left.\frac{dy}{dx}\right|_{t=0}=\frac{2\cdot 0}{4-3\cdot 0^2}=0$$

$(x,y)=(0,-1)$

$y-y_0=m(x-x_0)$

$y-{-1}=0(x-0)\longrightarrow y+1=0\longrightarrow y=-1$

$$\left.\frac{dy}{dx}\right|_{t=2}=\frac{2\cdot 2}{4-3\cdot 2^2}=\frac{4}{-8}=-\frac{1}{2}$$

$(x,y)=(0,3)$

$y-3=-\frac{1}{2}(x-0)$

$y-3=-\frac{1}{2}x$

DO YOURSELF

$t=-2$